

# Speed of sound in a Quark-Gluon-Plasma with one loop correction in mean-field potential

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## Abstract

We study thermodynamic properties and speed of sound in a free energy evolution of quark-gluon plasma (QGP) with one loop correction factor in the mean-field potential. The values of the thermodynamic properties like pressure, entropy and specific heat are calculated for a range of temperatures. The results agree with the recent lattice results. The speed of sound is found to be  $C_s^2 = 0.3$  independent of parameters used in the loop correction which matches almost with lattice calculations.

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## 1 Introduction

The matter formation during the early stage of universe expansion is considered to be a complicated phenomena. To solve this problem there are a number of experimental laboratories set up in the last two decades. It is believed that there is a phase transition during the early universe expansion to a phase of confining matter of hadrons from deconfined matter of free quarks and gluons [1, 2]. The process of transition is investigated in the experiments like relativistic heavy-ion collider (RHIC) at BNL and large hadron collider (LHC) at CERN. The study makes it attractive for the experimentalists and theorists to collaborate in this exciting field in the present day of heavy ion collider physics [3, 4]. In this short paper, we focus on the QGP evolution through the free energy expansion of the system with one loop correction. The free energy is constructed through a density of state which is obtained with a one loop corrected mean-field potential. So the free energy contribution from every constituent particles of the system is considered in this calculation. Moreover, the correction factor in the potential is obtained through coupling value, [5, 6, 7] and due to this correction value, there are changes in the free energy expansion of QGP fireball, and it also impacts in the stability of droplet with the variation of dynamical quark and gluon flow parameters  $(\gamma_q, \gamma_g)$ .

In this short article we calculate thermodynamic parameters like pressure, entropy and specific heat through the free energy. Then we calculate the speed of sound in a QGP with one loop correction in mean field potential which is extensively dealt with in our earlier paper [8]. The construction of density of states and the free energy with the loop correction has been explained in detail with these papers. In conclusion, we give the details of thermodynamic properties of QGP fireball like pressure, entropy, specific heat and speed of sound with the flow parameter  $\gamma_q = 8, \gamma_g = 8\gamma_q$ .

## 2 Density of states and free energy

The density of states in phase space with loop correction in the interacting potential is obtained through a generalized Thomas- Fermi model as [9, 10]:

$$\int \rho_{q,g} dq = \nu / \pi^2 [-V_{conf}(q)]^2 \frac{dV_{conf}}{dq}, \quad (1)$$

where,

$$V_{\text{conf}}(q) = \frac{2}{q} \sqrt{(1/\gamma_g)^2 + (1/\gamma_q)^2} g^2(q) T^2 [1 + g^2(q) a_1] - \frac{m_0^2}{2q}. \quad (2)$$

$\gamma_q = 1/8$  and  $\gamma_g = (8 - 10)$   $\gamma_q$  are quark and gluon flow parameters chosen in fitting the evolution of QGP droplets [8, 11, 12]. Then we finally get the density of state as:

$$\rho_{q,g}(q) = \frac{\nu}{\pi^2} \left[ \frac{\gamma_{q,g}^3 T^2}{2} \right]^3 g^6(q) A, \quad (3)$$

where

$$A = \left\{ 1 + \frac{g^2(q) a_1}{4\pi^2} \right\}^2 \left[ \frac{(1 + g^2(q) a_1 / (4\pi^2))}{q^4} + \frac{(2 + g^2(q) a_1 / \pi^2)}{q^2 (q^2 + \Lambda^2) \ln(1 + \frac{q^2}{\Lambda^2})} \right] \quad (4)$$

and  $\nu$  is the volume occupied by the QGP.  $q$  is the relativistic four-momentum in natural units.  $g^2(q) = 4\pi\alpha_s(q)$  where,  $\alpha_s(q)$  is the coupling value of quark and gluon with degree of freedom  $n_f$ , defined as

$$\alpha_s(q) = \frac{4\pi}{(33 - 2n_f) \ln(1 + q^2/\Lambda^2)}, \quad (5)$$

in which  $\Lambda$  is QCD parameter measured in the scale of lattice QCD and it is taken as 0.15 GeV. The coefficient  $a_1$  in the confining potential of the above expression is one loop correction in their interactions and it is given as [13]:

$$a_1 = 2.5833 - 0.2778 n_l, \quad (6)$$

where  $n_l$  is the number of light quark elements [14].

Now we calculate free energy for the system using the expression as in [15, 16]:

$$F_i = \mp T g_i \int dq \rho_{q,g}(q) \ln(1 \pm e^{-(\sqrt{m_i^2 + q^2})/T}), \quad (7)$$

with minimum energy cut off as:

$$V(q_{\min}) = (\gamma_{g,q} N^{\frac{1}{3}} T^2 \Lambda^4 / 2)^{1/4}, \quad (8)$$

where  $N = (4/3)[12\pi/(33 - 2n_f)]$ . The minimum cut off in the model makes the integral to have a finite value by avoiding the infra-red divergence while taking the magnitude of  $\Lambda$  and  $T$  as of the same order as in lattice QCD.  $g_i$  is the corresponding degeneracy factors for the subsequent free energies. The inter-facial energy obtained

through a scalar Weyl-surface in Ramanathan et al. [9, 17] with a suitable modification to take care of the hydrodynamic effects is given as:

$$F_{interface} = \frac{1}{4}\sqrt{(1/\gamma_q^2 + 1/\gamma_g^2)}R^2T^3. \quad (9)$$

The energy is used in replacing the role of bag energy of MIT model and it minimizes the drawback produced in numerical techniques by MIT model. The hadronic free energy is [18]

$$F_h = (g_i T / 2\pi^2) \nu \int_0^\infty q^2 dq \ln(1 - e^{-\sqrt{m_h^2 + q^2}/T}). \quad (10)$$

For quark free energy we consider quark masses  $m_u = m_d = 0 \text{ MeV}$  and  $m_s = 0.15 \text{ GeV}$  [15]. The hadronic particles are taken having the masses below 1.5 GeV and their contribution to the interaction is considered zero in nature. We consider these values as it has a dominant component in the contribution of the hadronized phase. We can thus compute the total modified free energy  $F_{total}$  as,

$$F_{total} = \sum_i F_i + F_{interface} + F_h, \quad (11)$$

where  $i$  stands for  $u$ ,  $d$  and  $s$  quark and gluon.

### 3 Thermodynamic properties and speed of sound

The thermodynamic properties like pressure, entropy and specific heat of the system can be calculated from the total free energy. The standard thermodynamics give the following relations [19]:

$$Pressure \ P = -\left(\frac{\partial F}{\partial v}\right) \quad (12)$$

$$Entropy \ S = -\left(\frac{\partial F}{\partial T}\right) \quad (13)$$

$$Specific \ heat \ C_v = T\left(\frac{\partial S}{\partial T}\right)_v \quad (14)$$

The behavior of entropy and specific heat with temperature indicate the nature of phase transition of the system. So, we calculate the entropy and specific heat for a flow parameter  $\gamma_q = 1/8$  and gluon parameter  $\gamma_g = 8\gamma$ . At this particular value we can get the stability of the droplet with maximum radius of  $r = 4.2 \text{ fm}$ . The stability is found more with increasing gluon flow parameter up to  $\gamma_g = 10\gamma_q$ . With the increasing gluon parameter the pressure of the system is

increased and very much effective in the free energy expansion, Yet the entropy and specific heat is not effective with the increasing gluon parameter. Now using these entropy and specific heat we calculate the speed of sound of QGP. The speed of sound is given as the ratio of these two thermodynamic parameters [20]:

$$\text{Speed of Sound } C_s^2 = \frac{S}{C_v} \quad (15)$$

The speed of sound is found to be almost constancy over the range of temperature and the model parameter.

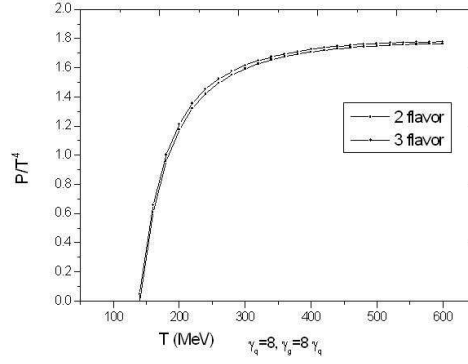


Figure 1: The variation of  $P/T^4$  vs.  $T$  at  $\gamma_q = 1/8$ ,  $\gamma_g = 8\gamma_q$  at the critical radius.

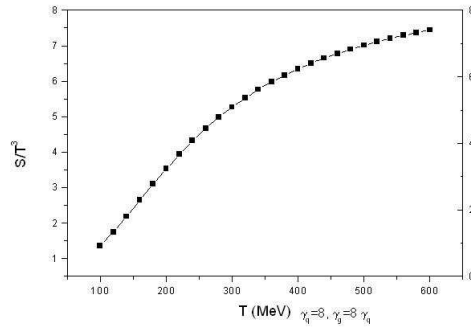


Figure 2: Entropy  $S/T^3$  vs.  $T$  at  $\gamma_q = 1/8$ ,  $\gamma_g = 8\gamma_q$  at the critical radius.

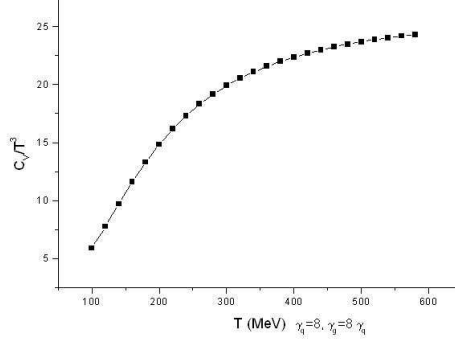


Figure 3: Specific heat  $C_v/T^3$  vs.  $T$  at  $\gamma_q = 1/8$ ,  $\gamma_g = 8\gamma_q$  at the critical radius.

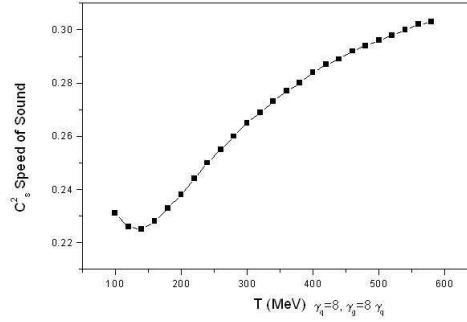


Figure 4: Speed of sound  $C_s^2$  vs.  $T$  at  $\gamma_q = 1/8$ ,  $\gamma_g = 8\gamma_q$  at the critical radius.

## 4 Results:

The thermodynamic properties like pressure, entropy and specific heat are calculated for quark and gluon flow parameter with the introduction of one loop correction factor in the interacting mean-field potential. They are calculated for a particular chosen flow parameter which fit well in the evolution of free energies with the stable largest droplet size. Free energy expansion can be produced with different stable droplet size with increasing gluon flow parameter up to  $\gamma_g = 10\gamma_q$ . There is change in the pressure but the parameter does not affect in the entropy and specific heat. The outputs from the calculation are shown in the figures. The figure 1 shows the variation of  $P/T^4$  with the temperature  $T$  for three flavor and two flavor of quarks. The variation in the figure determines the equation

of state (EOS) of the system. The equation of state is an essential component to understand the nature of QGP and to model the behavior of QGP in early universe formation. The variation agrees with the recent outcomes of lattice results with insignificant difference. In Fig.2 we show the plot of  $S/T^3$  with the temperature. The plot indicates the variation at lower temperature. It again shows the constancy over higher temperature. The result almost follows the lattice predictions for three and two flavors of quarks and it produced similar results with our earlier works without the loop correction [21]. Again Fig. 3 shows specific heat changes with the temperature. The specific heat is almost independent with higher temperature and the plot matches with the recent calculation of specific heat. Moreover, it is indicated by lattice QCD that formation of QGP droplets take place under the condition of roll-over phase rather than a sharp jump as temperature varies [22]. From the two thermodynamic parameters we use to calculate the speed of sound of the system. The result is shown in the figure 4 . From the figure, the speed of sound is well behaved in nature in terms of the lattice results of sound speed and almost the same to our earlier calculation of speed of sound without the loop correction. The result is approximately close to  $C_s^2 = 0.3$  which is obtained with this range of temperature. It is quite similar to the latest value of speed of sound of lattice QCD [23].

## 5 conclusion:

We conclude that due to the introduction of one loop correction in the mean field potential, we can describe the EOS through pressure and behavior of entropy and specific heat. So, we further obtained the speed of sound from these two thermodynamic parameters as  $C_s^2 = 0.3$  which is independent from the model parameter and temperature. It is a good result in comparison with recent lattice results.

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